A Complex Unified All-scale Potential for Positive and Negative Mass

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Abstract: The Standard Model faces challenges in its attempts to explain dark matter and dark energy. The Fractal Rings and Composite Elementary Particles Model (FRACEP) was developed as a possible alternative to shed some light on the problem. It is based on both positive and negative mass fundamental particles ($\mathcal{G}p$ and $\mathcal{G}n$ respectively), and it includes a fully-unified complex potential to characterize the behavior of these two mass sources. This potential is a function of mass and square-root of mass. It is real for positive mass sources, but complex for negative mass sources at every scale. The real component at the macro-scale far-field is consistent with Newton for positive and negative mass sources before a near-field transition to oscillation. The slightly out-of-phase oscillation could allow quasi-stable mixed-mass particles without the usual expected runaway repulsion between the positive and negative mass components. The closest-separation, near-field oscillation for the $\mathcal{G}n$-$\mathcal{G}p$ interaction quickly grows to the large repulsive level expected for a creation event – a condition that might have driven an inflationary expansion of space in the early universe. The potential's complex behavior might help explain some of the dark matter and energy puzzle.

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Keywords: negative mass, negative energy, alternate gravity, composite elementary particles, cosmic inflation.

MOTIVATION FOR THIS WORK

– Explore alternative views of the nature of space, the nature of the building blocks of the matter it contains, and how they interact on a fundamental level.
– The development of the FRACEP composite elementary particle model, with a fully unified potential, was the approach taken.

J. Giannini, "Fractal Composite Quarks and Leptons with Positive and Negative Mass". *Int. J. Mod. Theoretical Phys.*, 8 (**2019**): 41-63.

Th Standard Model is a remarkably successful model for describing the interactions of its elementary particles. With the discovery of a Higgs particle supporting the particle-mass-acquisition mechanism, the model is further accepted with increased credibility. Despite its success, there are still unresolved issues, notably at the cosmological level. Evidence supports the concept of Dark Matter halos around galaxies as a way of addressing the insufficiency of visible matter in the Universe. But, the nature of this Dark Matter is still unresolved. There are still no observed candidates of traditional positive-mass particles of unknown nature. There is also the question of the cause of expansion in the Universe and the change in the rate of the expansion – a matter still under debate. What exactly is the nature of the Dark Energy that is generally described as the source? Is Dark Energy an expression of the cosmological constant – or something else? Is Dark Energy somehow related to quintessence as a possible cause for expansion? Is there another possibility altogether?

This work was begun as a way of exploring alternative possibilities beyond the standard approach to viewing the nature of matter and its interactions, and the nature of space itself. It is a work in progress – with many questions, non-traditional views, and a mathematical basis yet to be fully defined.

A LITTLE BIT OF PHILOSOPHY

Chinese wisdom about the nature of the Universe says it consists of "being" and "non-being"

– both springing from the same source"

– "the Darkness within the Darkness, the gate to all mystery".

Lao Tzu, trans. Gia-Fu Feng and Jane English. "Verse 1". In *Tao Te Ching*; Vantage Books: NY, USA, verse 1, **1972**.

The Lao Tse (Tao-Te-Ching) tells the Chinese wisdom about the nature of the Universe: the "being" and the "non-being" where "these two spring from the same source" – "the Darkness within the Darkness, the gate to all mystery" [1]. Although Lao Tzu's words are ancient, they sound strangely reminiscent of the modern puzzle of dark matter and energy and their relation to the "Bright Universe" we see (the "being"). They hint of a "Dark Universe" we cannot see (the "non-being" – the unseen dark matter and energy within the darkness of the cosmos).

WHAT IS BEING AND NON-BEING
(a little bit of philosophy con't)

- **"Being" could be - the matter we see.**
- **Traditionally, mass can be seen or at least measured – by assumption, it is positive.**
- **"Non-being" could be the matter we don't see.**
- **Newton notes we gain information through observations - then applied generally. But, there are bodies beyond our senses' range (dark matter?)**

I. Newton. "Rules for the Study of Natural Philosophy". In *The Principia, Book 3,* trans. B. Cohn and A. Whitmann; U. California Press: Berkley, USA, **1999**.

About a thousand years after Lao Tsu, Newton [2] reflected on the fact that we gain information about matter through observations by our senses, and, that knowledge (of "sensible bodies") might then be applied universally to all bodies. But, he cautioned that there are bodies beyond the range of our senses, perhaps inferring the existence of the sub-atomic world that was not obvious. However, like the Tao-Te-Ching, his statement may be aptly prophetic about the possibility of negative matter.

A LITTLE BIT OF HISTORY

- **Newton interpreted mass as attracting bodies.**
- **General Relativity (1920s) reinforced that interpretation, adding the cosmological constant.**
- **Cosmological models (late 1930s) indicated an expanding universe needing "dark matter".**
- **Milgram (1983) modified Newton's laws (MOND) to address the problem.**
- **Perlmutter (1997) showed accelerated expansion.**

· F. Zwicky, "On the Masses of Nebulae and of Clusters of Nebulae". *Astrophys. J.*, 86 (**1937**): 217-246

· M. Milgram, "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis". *Astrophys. J.*, 270 (**1983**): 365-371.

· S. Perlmutter et al., "Measurements of the Cosmological Parameters Ω and Λ from the First Seven Supernovae at $z \geq 0.35$". *Astrophys. J.*, 483 (**1997**): 565-581.

Newton's theory of gravitation is remarkably successful at characterizing the behavior of attracting bodies from every-day scales on Earth to planetary scales, but with some limitations. In the early 1920s, Einstein's general theory of relativity changed the way gravitational interactions were viewed – from the action-at-a-distance pull between two mass-containing bodies to a field-oriented view based on the warping of space by the presence of the masses. General relativity resolved Newton's limitations and more – providing the basis of modern day cosmology. The cosmological constant (used by Einstein to counteract expansion) allowed a model of the expanding universe with the first concept of dark energy (DE). In 1997, Perlmutter et al. [3] presented evidence for accelerating expansion in the universe that was assumed to be the result of DE. There are a variety of candidates for DE, but, the nature of that phenomenon is still unclear [4].

In the late 1930's, cosmological models discovered a problem with their estimates of the amount of mass contained in the universe (a determinant of the expansion state). It was found that insufficient observed visible mass, implied many systems that appeared stable should be gravitationally unbounded [5]. The proposed solution to the missing mass problem was the hypothesis of dark (unseen) matter (DM). Although the nature of DM was unknown, increasing galaxy and nebula masses with this unseen matter, allowed the theory and observations to agree. Burbidge [6] cautioned that the results of this addition were strongly dependent on the assumptions used in the theory.

In 1983, Milgram proposed a modification to Newton's laws (MOND) that allowed the rotation rates (for some galaxies) to be satisfied without DM [7] – a theory that is still hotly contested. The predominating consensus of cosmologists, today, is the DM and DE paradigm [8]. The contents of the universe by this paradigm are divided as: DE (dark energy, identity unknown), ~73%; DM (identity unknown), ~23%; other non-luminous matter (gasses, neutrinos and super massive black holes) and luminous matter (stars, gasses and radiation), ~4%. Numerous possible candidates have been proposed for DM [9]. WIMPS and axions are the most favored ones, though, to date, the search for WIMPS has not been successful and some circles are pinning hopes on axion detection [10]. There is a growing interest in primordial black holes as a possible source of DM [11-12], as well as, the possibility of identifying DM at Fermi scales [13-14]. Chardin and Manfredi [15] considered the symmetric matter-anti-matter Dirac-Milne universe as an analog of the electron-hole system in a semiconductor as a possible relation of DM to negative mass. Barghout considers modifications to Newtonian dynamics that manifest themselves as DM and DE [16, 17].

MUST MASS BE ONLY POSITIVE (What Do the Models Say?)

- Newton, Schrodinger, and Dirac don’t require it.
- de Sitter’s solution (1917) to Einstein’s equations showed universe expansion without mass. But equal amounts of positive and negative mass make zero mass – the “empty universe solution”.
- Hoyle (2000) developed scale-invariant equations giving creation events with equal amounts of positive and negative mass.

J. Giannini. “Brief Survey of Quantum Models and Possible Negative Mass Solutions”, Researchgate, #349255496 (**2021**): 20 pages.
W. deSitter. “On Einstein’s Theory of Gravitation and Its Astronomical Consequences”, *Monthly Notices of the Royal Astronomical Society*, 78 (**1917**):3-28.
F. Hoyle, G. Burbidge, and J.V. Narlikai, *A Different Approach to Cosmology*; Cambridge U. Press: Cambridge, UK, **2000**.

The concept of negative mass has been considered as early as the late 1800s [18]. More recently it has been studied in in the context of General Relativity [19-20], and cosmological models showing equal and opposite mass particle creation [21].

Schrödinger’s relativistic wave equation is consistent with special relativity where the total energy (E^2) is a function of rest mass $(mc^2)^2$. The positive sign on the square root gives the positive energy solution, and the negative sign gives the negative energy solution, requiring a change in the interpretation of the wave function solution. In 1928, Dirac[22] presented an operator approach for the relativistic quantum mechanics consistent with relativity. Both the Schrödinger and Dirac relativistic equations allow solutions when the particle has negative kinetic energy and negative rest mass. These solutions correspond to the negative square root of the classical energy equation. However, the negative energy solutions, in the case of Dirac, are not ignored as they are in the classical case since nothing prohibits a charged particle from making a radiation transfer from a positive energy to a negative energy state. Unfortunately, the well recognized dual solutions imply the possibility of an electron spontaneously jumping between positive and negative energy states. Dirac addressed the interpretation of the negative energy solutions with the development of his Hole Theory which includes the concept of a sea of negative energy electrons. Feynman [23] formalized this by reinterpreting Dirac’s negative-energy solutions as representing the electrons moving backward in time giving them a reversed (positive) electric charge.

DUAL POSITIVE AND NEGATIVE MASS UNIVERSES

- **Chang's Dirac-extended model has four elementary matter types: positive matter & anti-matter, and negative matter & anti-matter.**
- **Einstein-based Janus has positive and negative elementary masses with different light speeds.**
- **FRACEP has composite elementary particles in both "Bright" and "Dark" Universes.**

Yi-Fang Chang, "Negative Matter as Dark Matter, and Its Judgment Test and Calculation of Ratio". *Int. J. Mod. Applied Phys.*, 9(1) (**2019**): 1-12.

J.P. Petit and G. D'Agostini, "Cosmological Bimetric Model with Interacting Positive and Negative Masses and Two Different Speeds of Light, in Agreement with the Observed Acceleration of the Universe". *Mod. Phys. Lett.* A29 (**2014**) 1450182 (15 pages).

J. Giannini, "Fractal Composite Quarks and Leptons with Positive and Negative Mass". *Int. J. Mod. Theoretical Phys.*, 8 (**2019**): 41-63.

In addition, single-metric [24], double-metric [25], and heuristic [26] dual universe models show four sets of particles (rather than the two of the Standard Model) based on both positive and negative mass – positive mass particles and anti-particles, and negative mass particles and anti-particles. Experiments are beginning to address the negative-mass question [27-32].

In [24], Chang extended Dirac's theory to develop the field equations leading to four types of matter: positive matter, positive anti-matter, negative matter and negative anti-matter. He further considered the nature of negative matter as DM at cosmic scales and some possible tests of the concept [33, 34].

Petite and his colleagues [35] developed the Janus cosmological model of the universe which is based on Einstein's equations. However, it considers the interaction of positive and negative masses where the two types of matter have different light speed. Their twin-universe system of coupled equations shows like-mass-types attracting and unlike-mass-types repelling, and includes both positive-energy photons and negative-energy photons. In a non-steady state solution [36], they address the possibility of negative matter clumping, the nature of DE and its relation to expansion in the universe. As an alternative, the Fractal Rings and Composite Elementary Particles (FRACEP) model [26] was developed.

THE FRACEP MODEL

- **Fractal Rings and Composite Elementary Particles**
- **Has composite quarks, leptons and bosons.**
- **"Bright Universe" matter and anti-matter has mostly positive mass. Anti-matter has slightly smaller mass value because of more negative mass.**
- **"Dark Universe" matter and anti-matter has mostly negative mass. Anti-matter has slightly larger mass value because of more positive mass.**
- **Complex unified potential describes interactions.**

J. Giannini, "Fractal Composite Quarks and Leptons with Positive and Negative Mass". *Int. J. Mod. Theoretical Phys.*, 8 (**2019**): 41-63.

J.A. Giannini, Fractal Rings and Composite Elementary Particle (FRACEP) Model: A Picture of Composite Standard Model Fundamental Particles. *Bull. Am. Phys. Soc.*, 61 (6) (**2016**), Session T1.031.

J.A. Giannini, "Feasibility of Constructing a Unified Positive and Negative Mass Potential". *Int. J. Mod. Theoretical Phys.*, 8 (**2019**): 1-16.

FRACEP provides composite versions of the Standard Model (SM) quarks, leptons and bosons (the "Bright Universe") having mostly positive mass. This model leads to an additional set of particles (the "Dark Universe") having mostly negative mass, that might provide some options in the DM search.

In FRACEP's dual universe, the mass-types interact like the Petit-model masses, and it contains four types of matter like the Chang model. Unlike the other models, FRACEP considers the mixed-mass-type internal structure of its particles, proposing an explanation of quark and lepton instability and half-life decay. The mathematical basis for FRACEP's construction is under development.

FRACEP's unified potential addresses Planck-length scales to the largest cosmic scales. It characterizes both positive-mass and negative-mass sources. In its most general form, it is a function of both time and space, $V_{FRACEP} = F(\mathrm{t}) \cdot V(r)$. The $F(\mathrm{t})$ describes temporal behavior important to the decay times of fermions, and the oscillation frequencies associated with charge and spin characteristics of FRACEP's charge-carriers and spin-carriers. This work considers only the spatial behavior ($V(r)$).

THE COMPLEX UNIFIED POTENTIAL

- Assumes Newton's 1/r is the 1st order term of a sine function
- Allows the sine argument to be multi-term, but requires it to be real, – the potential itself may be complex.
- Relaxes the requirement that $V(m, r) = -V(-m, r)$
- Requires the Unified Potential reduce to Newton at macro scales and, reduce to OPED nuclear potential at nuclear scales.

J.A. Giannini, "Feasibility of Constructing a Unified Positive and Negative Mass Potential". *Int. J. Mod. Theoretical Phys.*, 8 (**2019**): 1-16.

V_{FRACEP}'s functional form [37] was determined by empirically matching the behavior of V_N at macro scales, and the neutron-neutron scattering potential, in [38], at quantum scales. This characterizes positive-source masses to V_{FRACEP}'s smallest possible separation distance $r = 3.3\text{x}10^{-20}$ fm, the classical radius of its fundamental particles, $\mathcal{G}p$ and $\mathcal{G}n$.

The negative-source mass behavior, has some similarities, but also some differences from that of the positive source and is a direct result of the functional form needed to produce a unified function over all scales for the positive-source mass.

$$V_{FRACEP} = m_t\,[A_0(M) + B_0(\sqrt{M})]\,\sin[S(r, M) + T(r, \sqrt{M})]\,\exp(r, M).$$

$$A_0 = 1 / (0.18\,M), \qquad B_0 = 9.2095\times10^{-8}\,\sqrt{M},$$
$$E_0 = \exp\{-2.4\,r\,|M| / [M^2 + (m_{\mathcal{G}p}/m_\pi)]\,\};$$

$$S(r, M) = K_1 + K_f,$$
$$K_1 = -150\,(\pi/180)\,0.09\,(r/M)^2\,E_1, \qquad E_1 = \exp[-67\,(m_{\mathcal{G}p}/m_\pi) / |M|],$$
$$K_f = -(\pi/180)\,0.000092\,[M / 8\times10^{60}]^2\,[1.496\times10^{26}/\text{r}]^3;$$

$$T(r, \sqrt{M}) = K_2 + K_3 + K_4,$$
$$K_2 = -150\,(\pi/180)\,(0.00006/m_\pi)\,\sqrt{M} / r,$$
$$K_3 = 150\,(\pi/180)\,E_1 / \sqrt{M}, \qquad K_4 = K_3 / r.$$

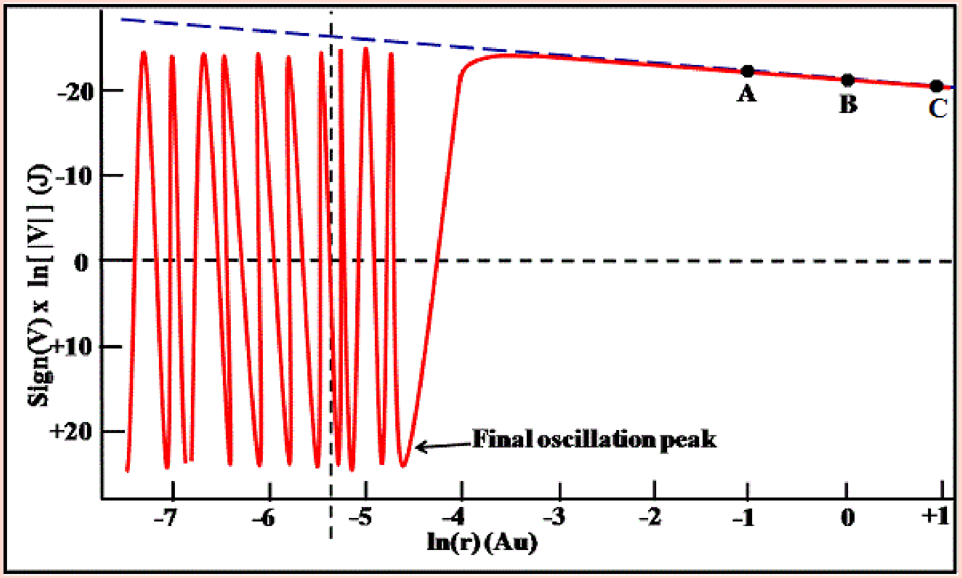
$M = m_s / m_\pi$ where m_s is the source mass for which the potential is computed, $m_\pi = 139.57$ MeV/c^2 (the mass of the pi-meson), and the m_t is the responding test mass, and $m_{\mathcal{G}p} = 1.72\times10^{-22}$ MeV/c^2 (the mass of the FRACEP positive mass fundamental particle).

$T(r, \sqrt{M})$ becomes imaginary when M is negative because of the $\sqrt{M}$ factor, giving V_{FRACEP} an imaginary component. This is not true of V_N which is always real, retaining the same function but changing sign. We assume an absolute value for M in the exponential function to guarantee V_{FRACEP} decays at large distances. For M < 0, the sine function is expressed as $\sin(S + i\, T) = [\sin(S) \cosh(T)] + [\cos(S)\, i \sinh(T)]$ giving

$$V_{FRACEP}\,(M < 0) = m_t \exp(r, M)\ \{[A_0 \sin(S) \cosh(T) - B_0 \cos(S) \sinh(T)] + i\, [A_0 \cos(S) \sinh(T) + B_0 \sin(S) \cosh(T)]\ \}$$

The result of this change is not a simple redistribution of the value of V_N between the real and imaginary parts of V_{FRACEP}. In nuclear physics, a complex potential is often interpreted as being related to the probability of the particle disappearing from observation [39] – that is, it is unstable. When expressed as $V = V_0 - i\,\Gamma$ where Γ is related to the decay time, $\tau = \hbar / 2\Gamma$.

MACRO SCALE BEHAVIOR FOR POSITIVE MASS



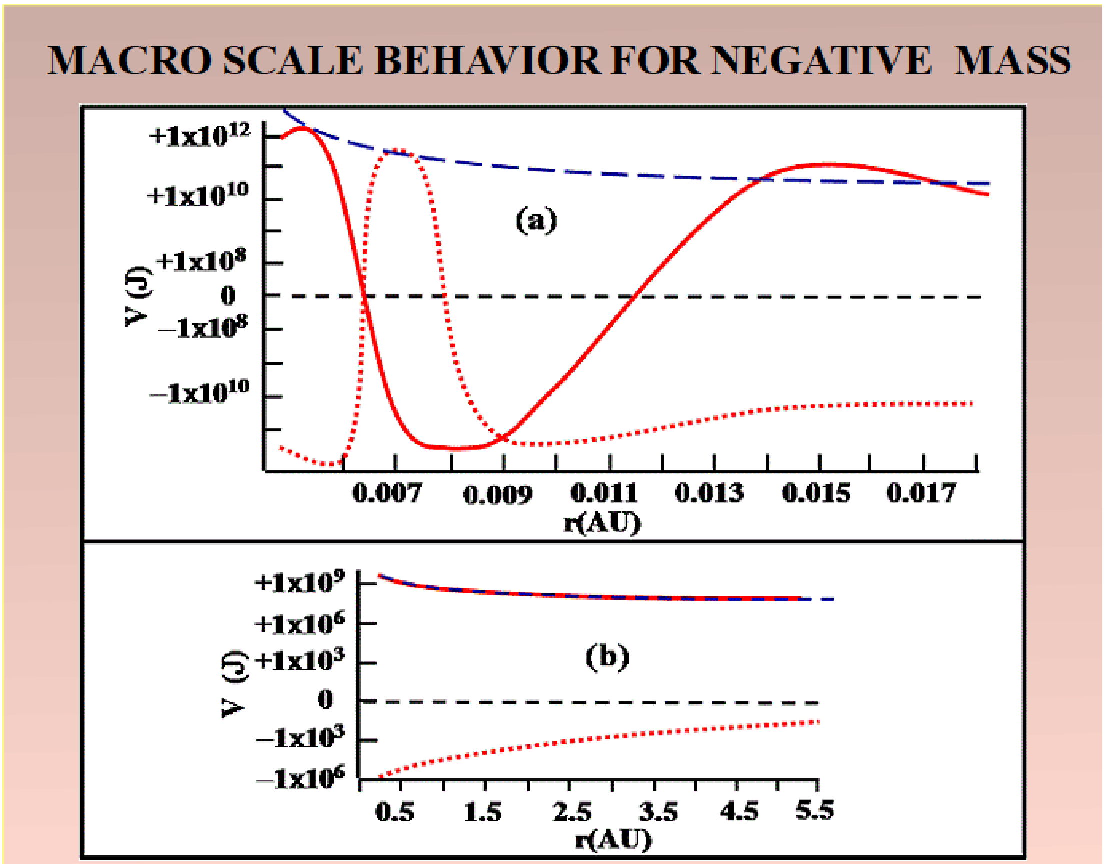
The solid red line is V_{FRACEP} and the broken blue line is V_N.

The macro-scale behavior of gravity is very wide-ranged, affecting everything from tiny masses at micrometer separations to large astronomical bodies at cosmic scales. V_{FRACEP} allows for the computation of the potential for both positive and negative source masses for all of these cases with a single multi-term function. At macro scales, for positive m_s, the potential has a zero imaginary part, while, the negative m_s has both real and imaginary parts. By comparison, the V_N is totally real regardless of the sign of the source.

The figure shows the characteristic behavior for all macro-scale positive-mass sources. The solid red line is V_{FRACEP} and the broken blue line is V_N. The vertical dashed line indicates the sun's radius, at 0.0047 AU. A indicates inside Mercury's orbit at 0.378 AU, B is Earth's orbit at 1 AU, and C is the asteroid Ceres orbit at 2.77 AU. (m_s = +1Su = 1.99×10^{30} kg, and m_t = 1 kg). In this case, m_s = +1 SU (the mass of the Sun) at solar system distances. The figure shows perfect agreement with V_N in the "main region" before beginning an oscillatory transition to quantum-like behavior (at the final oscillation peak) [37]. This is typical behavior for all positive source masses from for the smallest kg-sized masses through the largest cosmic masses. Note that all computations here assume approximately point sources; and, real world computations would require the usual application of finite-element-like techniques.

V is plotted in log-log format as the sign of V times the ln|V|. In the oscillatory region where V crosses from positive to negative values, the natural log introduces an artificial exponential growth to a singularity (between $V < -1$ and $V > +1$) that does not reflect the actual potential behavior. To address this and more properly reflect the behavior of V_{FRACEP}, the plot smoothly connects the two non-oscillatory regions on either side of the area in question.

V_{FRACEP} has the $1/r$ fall-off and the same sign as V_N (for $M > 0$ and for the real part when $M < 0$) at macro scales in the "main region" where m_s and m_t are sufficiently separated. In the "near-field transition region", where m_s and m_t are much closer together, V_{FRACEP} shows a non-Newtonian oscillation for both positive and negative sources.



The solid red line is V_{FRACEP} and the broken blue line is V_N.

For $M < 0$, the imaginary part generally decays more quickly at the larger r, and transitions to oscillation at smaller r before the real part. At the smallest macro-scale masses, this latter region corresponds to a transition from gravity-like to quantum-like behavior. Similarly, at the largest masses (e.g., for black hole-type masses, and even for Sun-like masses), the near-field region shows quantum-like behavior as well. The non-synchronized oscillation in V_{FRACEP}'s two components might result in a negative tidal-like force (as described by Barghout [16]) giving a longer range effect at galactic scales than in immediately obvious. This possibility requires further study.

This figure shows the characteristic behavior for all macro-scale negative-mass sources. The solid red line is the real part, V_{FRACEP}-R, and the dotted red line is the imaginary part, V_{FRACEP}-*I*. The broken blue line is V_N. At smaller *r* (a), both V_{FRACEP}-*R* and V_{FRACEP}-*I* oscillate (like the positive source potential, in the figure above. At larger *r* (b), V_{FRACEP}-*R* equals V_N, but, V_{FRACEP}-*I* changes sign, and is only ~0.05% of V_N at 0.378 AU. It decreases to ~0.008% of V_N by Earth's orbit. In this case, before going into oscillation, V_{FRACEP}-*R* is repulsive (like V_N) while V_{FRACEP}-*I* is attractive. ($m_s = -1$Su, and $m_t = +1$ kg).

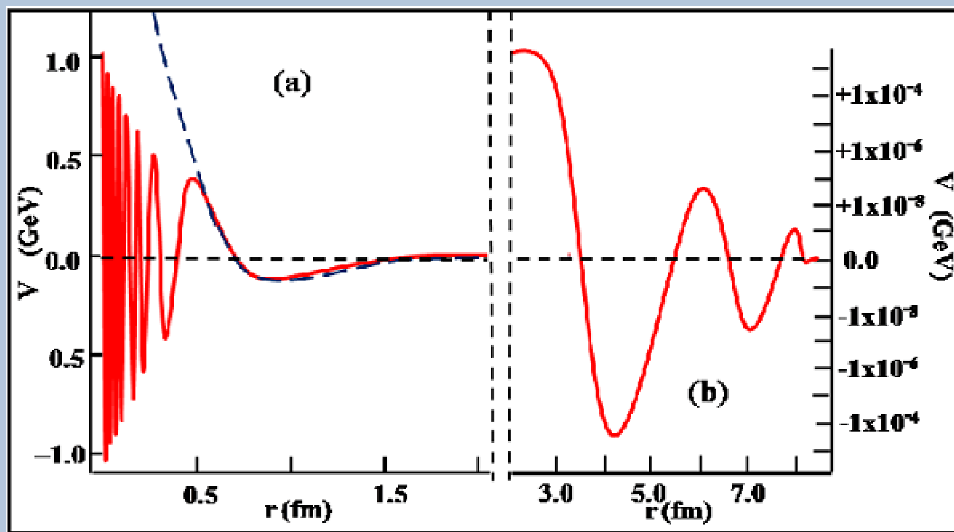
Here, $m_s = -1$SU at solar system distances, and the figure shows the real part (V_{FRACEP}-*R*) and the imaginary part *(*V_{FRACEP}-*I*) compared to V_N. For the "near-field transition region" (a), both V_{FRACEP}-*R* and V_{FRACEP}-*I* have a decaying oscillation not seen in V_N, but approximately bounded by V_N. For the "main region" (b), V_{FRACEP}-*R* agrees exactly in amplitude and sign (repulsive) with V_N before showing signs of beginning to diverge at the smallest *r*. V_{FRACEP}-*I*, on the other hand, has flipped sign (becoming attractive) and has a smaller amplitude, but a more rapid fall-off. This behavior is typical for all macro-scale masses – though the details of the fall-off rate and relative amplitudes vary.

Mass	**+55 kg**	**–55 kg**	**+2x10⁶ SU**	**–2x10⁶ SU**
Near-field Transition Region				
Range	5×10^{-16} to 5×10^{-4}	5×10^{-8} to 4×10^{-4}	1×10^{-5} to 1.0	1×10^{-5} to 1.0
V_N(J)	-10^{7} to -10^{-5} 1/r decay	10^{-1} to 10^{-5} 1/r decay	-10^{15} to -10^{10} 1/r decay	10^{15} to 10^{10} 1/r decay
V_F(J) Real	$\pm10^{-4}$ Oscillating	10^{141} to 10^{-5} Decay to 1/r by 4×10^{-5}	$\pm10^{14}$ to -10^{10} Oscillating decay to 1/r by 10^{-1}	$\pm10^{34}$ to $+10^{10}$ Oscillating decay to 1/r by 6×10^{-3}
V_F(J) Imag	0.0	-10^{132} to -10^{-23} Non-1/r decay	0.0	$\pm10^{34}$ to $\pm10^{6}$ Oscillating. decay
Main Region				
Range	5×10^{-5} to $9\times10^{+3}$	5×10^{-5} to $9\times10^{+3}$	1.0 to 9×10^{4}	1.0 to 9×10^{4}
V_N(J)	-10^{-4} to -10^{-13} 1/r decay	10^{-4} to 10^{-13} 1/r decay	-10^{10} to -10^{5} 1/r decay	10^{10} to 10^{5} 1/r decay
V_F(J) Real	-10^{-4} to -10^{-13} 1/r decay	10^{-4} to 10^{-13} 1/r decay	-10^{10} to -10^{5} 1/r decay	10^{10} to 10^{5} 1/r decay
V_F(J) Imag	0.0	-10^{-20} to -10^{-31} Non-1/r decay	0.0	-10^{6} to -10^{-9} Non-1/r decay

For comparison, the table summarizes the variations for a larger and a smaller mass. This summarizes results for V_{FRACEP} at two macro-scale source masses bracketing the solar system results in the above two figures (that is, for both positive m_s and negative

m_s). V_N in each region is given for comparison. For the 55 kg cases, range is in meters; and, for the 2x10[6] SU, range is in light-years.

QUANTUM SCALE BEHAVIOR FOR POSITIVE MASS

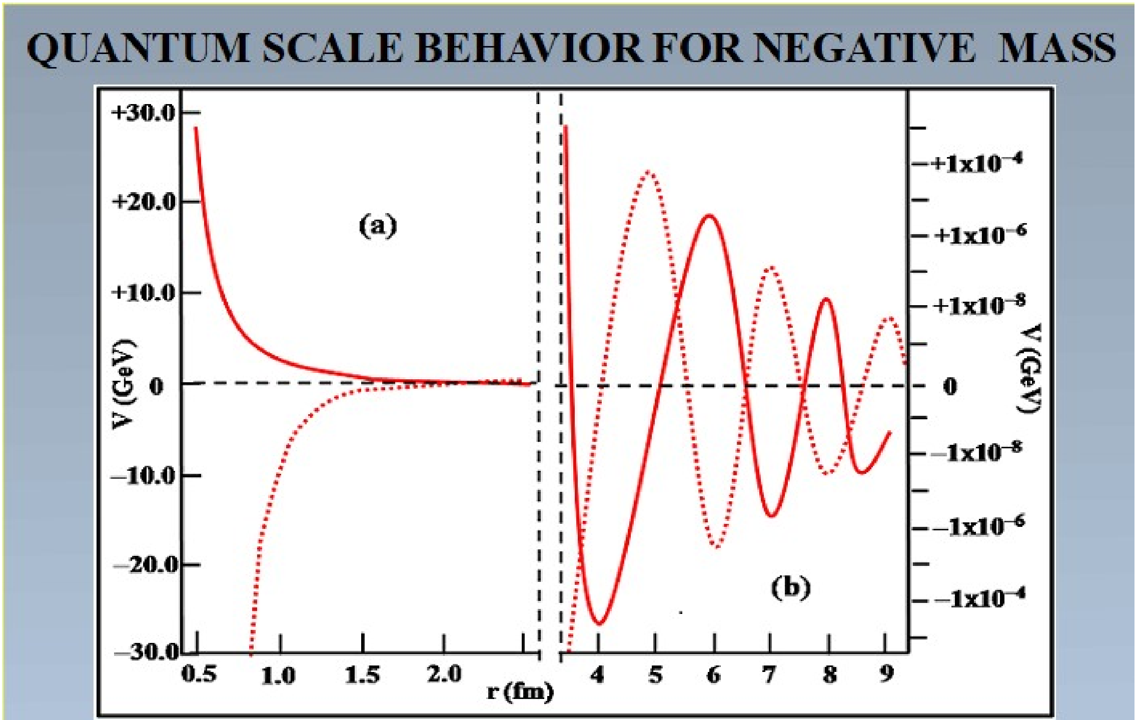


The solid red line is V_{FRACEP} and the broken blue line is OPED

At quantum scales, V_{FRACEP} characterizes both positive m_s and negative m_s. Like the macro-scale cases, V_{FRACEP} for positive m_s is totally real, while for negative m_s, it has both real and imaginary parts. OPED is totally real. With decreasing r below ~0.5 fm, V_{FRACEP} remains finite – oscillating with increasing frequency as it approaches maximum amplitude, while OPED continues to increase (a region where OPED is not valid). For $r \geq 2$ in the right curve where OPED is also not valid, the oscillating tail exponentially decreases. This is characteristic for all sources to the smallest mass.

The figure shows the characteristic behavior at the quantum scale. The solid red line is V_{FRACEP} and the broken blue line is OPED. There are no values for OPED in the lower (smaller r, near-field) or upper (larger r, far-field) tail regions. The r-values between ~0.5 and 2.0 are the usual values for OPED for the pi-meson interaction ranges. The V_{FRACEP}'s near-field oscillation ($r < 0.25$) approaches its maximum amplitude ($\pm$1.234 GeV) where OPED increases exponentially.

The V_{FRACEP}'s damped oscillation in the far-field ($r > 2.5$) is shown on an exaggerated scale. ($m_s = m_t = m_\pi$). In this case with the pi-meson source, V_{FRACEP} shows reasonable agreement with OPED (<20% RMS) within the usual range for the pi-meson interactions (~0.3 fm to ~2.5 fm) – in the left curve. Here, OPED has a spring-like oscillation that is pegged at the lower end, and, stretched and damped as r increases (a behavior well matched by V_{FRACEP}).



The solid red line is V_{FRACEP} and the broken blue line is OPED

This figure shows the corresponding behavior for the negative pi-meson source. The solid red line is the real part, V_{FRACEP}-*R*; and the dotted red line is the imaginary part, V_{FRACEP}-*I*. The amplitude for the leading tail (a), corresponding to the very near-field in the left plot in the positive mass figure above, has increased so rapidly that there is no obvious oscillation at the smallest *r* where both the real and imaginary parts increase exponentially due to the hyperbolic sine and hyperbolic cosine functions. Here, for *r* where the OPED model is valid for $+m_\pi$, both V_{FRACEP}-*R* and V_{FRACEP}-*I* show no oscillation at smaller *r*. In this case, V_{FRACEP}-*R* is repulsive while V_{FRACEP}-*I* is attractive before going into the larger *r* oscillation (b), similar to the $+m_\pi$ case. ($m_s = -m_\pi = -139.57$ MeV/c^2 and $m_t = +m_\pi$).

For the real part, the potential is positive and repulsive, while the imaginary part is negative and attractive. As *r* increases to the far-field, corresponding to the right plot in positive mass figure above, the oscillating tail exponentially decays. Like the macro-scale cases, the real and imaginary parts are not synchronized and are often out of phase. The transition region between near and far field is so narrow that it is not shown here. (OPED does not address negative mass).

CONCLUSIONS

- **Replacing 1/r with a sine function with a multi-term argument provided a unified potential that characterizes macro-scales to quantum-scales for both positive and negative mass sources.**
- **The negative mass realization of the potential is complex, while the positive mass realization is purely real.**
- **The positive mass potential remained bounded, while the negative mass potential in the oscillatory region was not.**

This effort produced a unified potential (V_{FRACEP}) that characterizes quantum scales through macro and cosmic scales. As developed, it was not intended to provide a model of specific phenomena – only an operational characterization of the overall field behavior – for both positive m_s and negative m_s.

The existence of negative mass is an integral part of V_{FRACEP}. The ultimate goal of this work is a description of the fundamental scales potential relevant to the FRACEP composite particles which includes both positive and negative mass elements. The imaginary part of the negative-mass characterization in V_{FRACEP} comes from the terms with the square-root of the source mass – an effect not found in V_N or OPED. An intuitive concept for negative mass (and the imaginary part of its potential) is uncertain at this time.

At macro scales, for $M > 0$, V_{FRACEP} leads to an attractive potential that agrees with V_N to <0.001% in the "main region" before going into the oscillatory behavior at smaller r that could be interpreted as quantum-like effects around otherwise macro-scale or cosmic-scale sources. For $M < 0$, the real part of V_{FRACEP} leads to a repulsive force (like V_N). The corresponding imaginary part is attractive, and, both parts oscillate (unlike V_N which does not). The full implications of this behavior, relative to cosmological dark matter, have yet to be determined.

Bright matter (positive mass baryons) constitutes <5% of all of the matter/energy in the universe. Dark matter (non-baryonic particles) and dark energy compose the rest. Models generally assume V_N behavior with luminous matter; but, galactic dynamics and other measures indicate there is not enough matter to account for the observations. The search for positive-mass non-traditional matter or modified Newtonian dynamics has not given completely satisfying answers to the dilemma. FRACEP's negative mass concept may help.

At quantum scales for $M > 0$, V_{FRACEP} agrees reasonably well with OPED within the accepted valid range. For $M < 0$, V_{FRACEP} seems to imply any negative m_s interaction is maintained at a much greater distance than observed for positive m_s. The implications of this have not been determined.

CONCLUSIONS (con't)

(work in progress)

- **A Big Bang creation event may reflect positive mass - negative mass repulsion at closest approach leading to inflationary expansion.**
- **Quantum-scale oscillation mismatch between positive and negative mass potentials may provide a mechanism allowing quasi-stable mixed-mass composite elementary particles.**
- **Quantum-scale potential oscillations may provide a mechanism allowing stable same-mass composite elementary particles.**

This work shows there is a possibility that the $1/r$-Newtonian potential might represent a first order approximation to a more complicated function that unifies all the scales. V_{FRACEP} has characteristic behavior that is consistent with bright matter (positive mass) observations. But, it also allows for the characterization of non-traditional (negative mass) matter that may illuminate the puzzle of dark matter and energy – possibly including things like dark halos around galaxies and the expansion characteristic of the universe.

One possible point of interest comes from the work of Hoyle, et al. in their development of a canonical form of gravity equations that reduce to General Relativity under certain conditions [40]. Their theory indicates that during a creation event (like the

Big Bang), pairs of particles, one with positive mass and the other with negative mass, are created at energies ~6x10^{18} GeV.

Both the real and imaginary parts of V_{FRACEP} for $m_s = \varsigma n$ (FRACEP's negative mass fundamental particle) have approximately that magnitude at $r \sim 3\text{x}10^{-20}$ fm. This might imply that a pair of particles is created approximately in contact (since the classical radius of the two particles is approximately 3x10^{-20} fm), and, there is an explosive repulsive force between them at that time. One might speculate that the explosive force (driven by the repulsive potential – maybe the initial Dark Energy) is what fed the initial expansion of the universe at the moment of the Big Bang. Further investigation of this possibility is needed.

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